

Collective dynamics in systems of active Brownian particles with dissipative interactions

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We use computer simulations to study the onset of collective motion in systems of interacting active particles. Our model is a swarm of active Brownian particles with internal energy depot and interactions inspired by the dissipative particle dynamics method, imposing pairwise friction force on the nearest neighbors. We study orientational ordering in a 2D system as a function of energy influx rate and particle density. The model demonstrates a transition into the ordered state on increasing the particle density and increasing the input power. Although both the alignment mechanism and the character of individual motion in our model differ from those in the well-studied Vicsek model, it demonstrates identical statistical properties and phase behavior.

I. INTRODUCTION

Dynamic self-organisation and, in particular, mechanisms of swarming behavior of microorganisms, cells, and animals remain one of the most intriguing problems at the interface of physics and biology. Numerous physical models of interacting self-propelled particles have been proposed recently to study these phenomena (see review papers [1–4]). All these models capture the essential prerequisites for swarming: out-of-equilibrium state, which is manifested in the self-propulsion of particles or other mechanisms of transforming external energy into directed motion, and aligning or attractive interactions between the particles. The motion of individuals has been described in the simplest case by particles moving with a constant speed and subjected to angular noise [5–7]. More advanced presentations of active agents can include friction, thrust force, and noise, like, for example, the active Brownian particle model (ABP) [8], or even a very detailed mechanics of cell or animal locomotion [9, 10]. The interactions required for the transition from individual to collective dynamics have been introduced in a variety of ways. In the standard Vicsek model, the swarming results from the action of a collision-type interaction that aligns the velocity of each actively moving particle in a big ensemble to the average local velocity [5, 6]. Alternatively, the particles' individual direction of motion can be coupled to the mean orientation or position of the swarm [11–14]. In some implementations, the type of many-body interaction depends on the distance between neighbouring particles [15].

The ABP model, based on the Langevin equation for particle velocity, has received much attention in literature and has been successfully applied to a variety of problems [3]. Depending on the type of system under study, different types of coupling between the ABPs been used. Several realisations of the model assumed only conservative interactions [8, 16] or chemical interactions [17, 18] between moving agents. Another development was based on the theory of canonical-dissipative system [19]. Erdmann and Ebeling studied the active Brownian particle

model with Oseen-type hydrodynamic interactions [20] and observed several swarming modes. Recently, Grossmann et al. [21] studied the onset of the collective motion in a system of ABPs with velocity alignment and both passive and active fluctuations and found not only orientational order-disorder transition but also bistable dynamics states. Swimming particles with hydrodynamic interactions were studied using the mesoscale simulation methods or direct solution of the Stokes equation also showed an onset of collective dynamics [22, 23].

The special attention paid to hydrodynamic effects in these studies is not accidental: the language of hydrodynamics is conceptually well suited for description of the swarm motion. The analogy has been explored already in the early papers by Toner and Tu [1, 2, 24, 25], who introduced a Navier-Stokes-like continuum model for active materials. The hydrodynamic behavior of active swarms can be inferred directly from microscopic description. Bertin *et al.* derived hydrodynamic equations governing the density and velocity fields from the microscopic dynamics for a gas of self-propelled particles with binary interactions [26]. One can notice that similar ideas are exploited in the mesoscale methods in fluid modelling such as multi-particle collision dynamics (MPCD) [27, 28], where a collision operator is used to align particles to the average local flow direction, or the dissipative particle dynamics (DPD) [29, 30], where the hydrodynamics comes in through inelastic collisions between the particles. Both of these methods designed to respect the momentum transport (long-wave hydrodynamic modes) and to punish the fluctuations (the high frequency modes) to achieve the hydrodynamic behavior at longer time and lengthscales. Obviously, the swarming behavior can be achieved through MPCD or DPD-like interactions as well. In fact, the possibility to develop the collective dynamics through dissipative interactions has been recently investigated by Grossman et al. [31]. In their model, the system of active particles with spring-dashpot interactions demonstrated a discontinuous transition into the aligned state upon reduction of noise and various types of collective migration or vortex-like motion

depending on the confinement. Thus, we are convinced that a system with dissipative interactions can have at least qualitatively similar dynamics to the models with aligning interactions like the Vicsek model. In the spirit of these observations, it is tempting to test whether the quantitative features of the swarming behavior can be reproduced in a generic dissipative model.

In this paper, we study the dynamics of such a model and demonstrate that collective motion regimes can be achieved in the same way as in the standard Vicsek model. We combine two well-developed approaches: the active Brownian particle model, which allows us to introduce the self-propulsion and interactions with the environment in a transparent way, and dissipative interactions for the active particles, so that the collective dynamics would arise from explicit pairwise forces. By analyzing the statistical properties of this hybrid model, we show that it has the same universal properties across the order-disorder transition as those reported for the Vicsek model despite the big differences in the implementation. In Section II we describe the construction of the model, in Section III we show its statistical properties in a wide range of parameters, calculate the phase boundary for the orientational order-disorder transition. We discuss the results in Section IV, and then conclude the paper in Section V.

II. MODEL AND SIMULATION SETTINGS

To study the dynamic self-organisation of active particles we introduce a two level model. At the single particle level we include the factors determining the particle motion in a thermal bath (such as a water reservoir): bath temperature and thermal noise/fluctuations, friction, and a motor. At the higher level, we introduce dissipative interparticle interactions. At the two-body level, we also include some noise, whose nature is, however, different from that of the environment. The noise at the many-body level refers to biomimetic behavioral features like imperfect alignment of particles to their neighbours or sudden voluntary switching of the direction of motion. Although the noise in the interaction is of non-thermal origin, it can also be characterized by some effective temperature. We can show that the collective behavior can be associated with these temperatures.

A. Equation of motion of a single self-propelled particle

At the single particle level, we follow the ABP model [8]. Here, we consider the motion in two dimensions. The equation of motion of an individual particle i is the Langevin equation for the velocity with an added thrust term

$$m \frac{d\mathbf{V}_i}{dt} = -\gamma^E \mathbf{V}_i + \mathbf{F}_i^T + \sqrt{2D^E} \boldsymbol{\xi}_i(t) \quad (1)$$

For simplicity, we will always set the particle mass to unity. The first two terms in Eq. (1) are standard Langevin friction and random force, where γ^E is the coefficient of viscous friction, which is set by the properties of the *environment*. The last term is the random force of strength D^E and $\boldsymbol{\xi}(t)$ is representing a Gaussian white noise with zero-mean and unit variance. The strength of the noise is set by the fluctuation-dissipation relation at the ambient temperature T^E

$$D^E = \frac{k_B T^E}{\gamma^E}. \quad (2)$$

We should stress that the fluctuations introduced in Eq. (1) do not depend on the particle's speed or direction of motion and therefore can be referred to as *passive* [21].

The thrust term in the depot model has the form:

$$\mathbf{F}_i^T = \frac{dq}{c + dV_i^2} \mathbf{V}_i \quad (3)$$

where d is the constant determining the rate of conversion of internal energy of the active agent into kinetic energy, c is the parameter controlling energy loss, and q is the constant determining the gain of energy from the environment. The steady state motion of the active particles is characterized by the velocity $V_0^2 = V_{0x}^2 + V_{0y}^2$, which is expressed through the system's parameters as

$$V_0^2 = \frac{q}{\gamma^E} - \frac{c}{d} \quad (4)$$

at $q > \gamma^E c/d$ [3, 8]. The steady state velocity distribution for various q is shown in Fig. 1. At $q = 0$ we observe the Maxwell distribution of the velocities corresponding to the systems's temperature T^E , while at non-zero energy influx rates we see either a broadened distribution centered at zero (at $q < \gamma^E c/d$) or two bell-like peaks around the stationary velocity $V_{0x} = V_0/\sqrt{2}$ with the peak width controlled by the temperature. Thus, the model exhibits a transition from dissipative to driven regime upon increase of the energy conversion coefficient q [8, 16, 20]. In the driven regime, attractive and aligning interparticle interactions can lead to collective dynamics.

B. Interparticle interactions

The collective behavior is impossible in a system of non-interacting agents. While one can expect some swarming (particle clustering) already with isotropic central interactions, the global symmetry breaking and onset of directed transport requires that particle velocities are aligned. Here we should note that a spontaneous transition into a globally aligned state is impossible in equilibrium system with perfectly elastic collisions and without any dissipation due to conservation of the total linear momentum. The local alignment can be realised by different means, the best known example being the

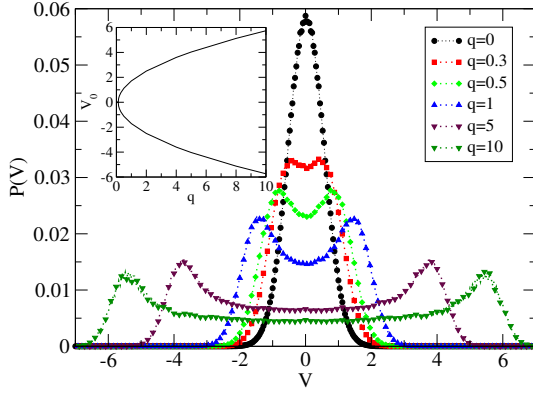


FIG. 1: Instantaneous 1D velocity distributions for the active Brownian particles at $c = 0.8$, $d = 2.0$, $T^E = 0.3$. *Inset*: Bifurcation diagram for the stationary velocity V_0 as a function of q .

Vicsek model, where particles are aligned to the local mean velocity field [5]. Another example of the aligning interaction is the hydrodynamic interaction of fluid molecules, solute particles, or swimmers [23]. At the microscopic level, the onset of hydrodynamic behavior is achieved by suppressing the relative motion of the neighbouring particles with a friction force and preserving the local mean velocity. As a result, the fluid quickly relaxes to the stationary state. This idea is realised in a number of mesoscale simulation methods, which are known to produce correct hydrodynamics: lattice Boltzmann (LB) method [32], MPCD, and DPD. In the LB and MPCD implementations the collisions are collective, similar to the Vicsek model, while in DPD the friction force is pairwise and is applied to each pair of colliding particles [33]. So, the latter method is ideally suited for our purpose as it presents a simple way to control the strength of the aligning interaction.

Here, we introduce a dissipative force between the ABPs in the same way as it is done in the DPD method. The total force $\mathbf{F}_i(t)$ acting on each particle is then given by:

$$\mathbf{F}_i = \mathbf{F}_i^S - \gamma^E \mathbf{V}_i + \mathbf{F}_i^T + \sqrt{2D^E} \boldsymbol{\xi}_i(t) \quad (5)$$

where \mathbf{F}_i^S is the force that comes from interactions within the swarm. \mathbf{F}_i^S consists of three parts:

$$\mathbf{F}_i^S = \sum_{j \neq i} (\mathbf{F}_{ij}^C + \mathbf{F}_{ij}^D + \mathbf{F}_{ij}^R) \quad (6)$$

where F_{ij}^C , F_{ij}^D , and F_{ij}^R represent the conservative, dissipative, and random forces between particles i and j , respectively. The conservative force that reflects the excluded volume interactions is defined as:

$$\mathbf{F}_{ij}^C = F^C(r_{ij}) \hat{\mathbf{r}}_{ij} \quad (7)$$

where $F^C(r)$ is a non-negative (repulsive) scalar function determining the distance dependence of the repulsion,

$\mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j$ is the distance between particles i and j , $r_{ij} = |\mathbf{r}_{ij}|$ is its magnitude, and $\hat{\mathbf{r}}_{ij} = \mathbf{r}_{ij}/r_{ij}$ is the unit vector from j to i . We choose $F^C(r)$ to describe a soft repulsion:

$$F^C(r) = \begin{cases} a \left(1 - \frac{r}{r_r}\right), & r \leq r_r \\ 0, & r > r_r \end{cases} \quad (8)$$

where a is a parameter determining the maximum repulsion between the particles, $r_r = r_c/2$ is the radius of the repulsion zone, and r_c is the cut-off distance.

The dissipative force punishes the velocity differences between the neighbouring particles and, therefore, provides a mechanism of relaxation of the velocity field toward the stationary state. We take it in the form of a friction force applied to the component of the motion in the direction of the particle connecting vector, i.e. a speed adjustment for particle moving together in the same direction.

$$\mathbf{F}_{ij}^D = -\gamma^S \omega^D(r_{ij}) (\hat{\mathbf{r}}_{ij} \cdot \mathbf{V}_{ij}) \hat{\mathbf{r}}_{ij} \quad (9)$$

Similarly, the friction can be applied to the motion perpendicular to the connecting vector [34], in which case it will predominantly act as an aligning interaction. In both cases, the parameter γ^S controls the dissipative strength of the interaction and by varying it we can accelerate or delay the alignment.

In a thermalized system, the stochastic force \mathbf{F}_{ij}^R compensates the loss of kinetic energy due to the dissipative force. It provides random "kicks" in the radial direction r_{ij} causing misalignment of particles' velocities.

$$\mathbf{F}_{ij}^R(t) = \sqrt{2D^S} \omega^R(r_{ij}) \xi_{ij}(t) \hat{\mathbf{r}}_{ij} \quad (10)$$

where D^S determines the strength of stochastic interactions, and $\xi_{ij}(t)$ is a random variable with a Gaussian distribution and unit variance. In hydrodynamic simulations, it is usually required that the noise ξ_{ij} is symmetric in ij , the kicks satisfy Newton's third law and conserve total momentum [35]. This requirement, however, can be omitted for active particles. A swarm temperature, T^S , can be defined via the fluctuation-dissipation relation

$$D^S = \frac{k_B T^S}{\gamma^S} \quad (11)$$

In Eqs. (9)-(10), $\omega^D(r)$ and $\omega^R(r)$ are weight functions, which for simplicity can be selected to be similar in form to the repulsive scalar function $F^C(r)$:

$$\omega^D(r) = [\omega^R(r)]^2 = \begin{cases} \left(1 - \frac{r}{r_c}\right)^2, & r \leq r_c \\ 0, & r_r < r < r_c \end{cases} \quad (12)$$

In this model, we can regulate the interparticle interaction by changing the effective temperature, T^S , and the friction coefficient, γ^S . The effective temperature determines the average degree of alignment the system can

tolerate, while the friction coefficient determines the dissipative power of a single collision and the speed of relaxation toward the stationary state. Note that in this case the friction and the noise depend on the particle relative position and velocities. The noise is generated by the individual itself so the fluctuations can be referred to as *active* [21]. Clearly, the global ordering should depend on both types of fluctuations, thermal (environmental) and non-thermal (collective). The whole set of the DPD terms reflects the behavioural contributions to the motion. For an animal or robotic systems it amounts to respecting the excluded volume and adjusting the motion to the neighbours. The stochastic term in this context plays a role of angular noise or imperfect alignment of the agents to their neighbours' direction of motion.

C. Simulation settings and motion statistics

We used a two-dimensional system with periodic boundary conditions. The primary box size was fixed at 200×200 units and we varied the number of particles in the main cell to set the number density ρ . Simulations were performed with time step of $\Delta t = 0.01$. Particles were propagated using the Verlet algorithm [36]:

$$\mathbf{r}_i(t + \Delta t) = 2\mathbf{r}_i(t) - \mathbf{r}_i(t - \Delta t) + \Delta t^2 \mathbf{F}_i(t) \quad (13)$$

The velocities of particles were calculated using Störmer-Verlet method:

$$\mathbf{V}_i(t) = \frac{\mathbf{r}_i(t + \Delta t) - \mathbf{r}_i(t - \Delta t)}{2\Delta t} \quad (14)$$

Total number of time steps in each run was set to 1×10^7 . All simulations were performed with the following set of key parameters: $r_c = 3$, $\gamma^S = 1$, $T^S = 0$, $\gamma^E = 0.3$, $T^E = 0.3$, $a = 1$, $d = 2$, $c = 0.8$.

To characterise the collective motion in our model we use three different correlation/distribution functions. The velocity autocorrelation function is calculated as

$$C(t) = \frac{1}{N} \left\langle \sum_{i=1}^N \frac{\mathbf{V}_i(0) \cdot \mathbf{V}_i(t)}{|\mathbf{V}_i(0)| \cdot |\mathbf{V}_i(t)|} \right\rangle \quad (15)$$

The radial distribution function is defined by

$$g(r) = \frac{L^2}{N(N-1)} \left\langle \sum_{i=1}^N \sum_{j \neq i}^N \delta(r - |\mathbf{r}_{ij}|) \right\rangle \quad (16)$$

where δ is the Dirac delta function, $\langle \cdot \rangle$ stands for the ensemble average. The two-point velocity correlation function is calculated as

$$C_{\parallel}(r) = \frac{1}{N(N-1)} \left\langle \sum_{i=1}^N \sum_{j \neq i}^N \frac{\mathbf{V}_i(t) \cdot \mathbf{V}_j(t)}{|\mathbf{V}_i(t)| \cdot |\mathbf{V}_j(t)|} \right\rangle \quad (17)$$

where i and j label particles separated by distance $r = |\mathbf{r}_{ij}|$. With this definition, two particles with parallel

(antiparallel) velocities give a correlation of $+1$ (-1). The angular brackets denote the ensemble average. To characterise the swarming behavior of the particles we also perform a cluster analysis. Cluster in our model is defined as a group of particles with a distance between neighbours smaller or equal to the cut-off radius r_c , therefore, particles interacting directly or via neighbouring agents are included into one cluster. We calculate the number of clusters and mean cluster size.

We characterise the orientational ordering by the polar order parameter, which quantifies the alignment of the particle motion to the average instantaneous velocity vector

$$\varphi(t) = \langle \cos \theta_i(t) \rangle = \frac{1}{N} \sum_{i=1}^N \frac{\mathbf{V}_i(t) \cdot \langle \mathbf{V}(t) \rangle}{|\mathbf{V}_i(t)| |\langle \mathbf{V}(t) \rangle|} \quad (18)$$

This order parameter has been extensively used to describe the orientational ordering in various systems of self-propelled particles [5, 37]. It turns zero in the isotropic phase and finite positive values in the ordered phase, which makes it easy to detect the transition.

To locate transition points precisely we also calculated the Binder cumulant [38]

$$G_L = 1 - \frac{\langle \varphi_L^4 \rangle_t}{3 \langle \varphi_L^2 \rangle_t^2} \quad (19)$$

where $\langle \cdot \rangle_t$ stands for the time average and L denotes the value calculated in a system of size L . The most important property of the Binder cumulant is a very weak dependence on the system size so G_L takes a universal value at the critical point, which can be found as the intersection of all the curves G_L obtained at different system sizes L [37] at fixed density. To detect the transition points in $q - \rho$ plane precisely we plot three curves for different L at constant density and find the point where they cross each other. Then, we use those points to construct the phase diagram.

III. RESULTS

A. Collective motion

We will illustrate the collective dynamics in our model by sequentially changing one of the two main parameters: the density ρ and the parameter controlling energy influx rate, q , which therefore determines the average propulsion speed of the particles. In Fig. 2 we display simulation snapshots obtained at a fixed input power $q = 0.3$ and different particle number densities. All of the snapshots feature significant density fluctuations. There is no obvious global ordering in the system but we can detect an increase of the cluster size. At fixed power q , an increase of the particle number density ρ leads to stronger density fluctuations and the velocity alignment. In Fig. 2(b),(c), we can notice a formation of dense particle groups at high concentrations, which move roughly in

the same direction. At low density (Fig. 2(a)), however, particles cannot form large groups and their velocities distributed randomly.

Figure 3 illustrates the variation of the statistical characteristics of the swarm upon a change of the particle concentration. The velocity autocorrelation function, $C(t)$, in Fig. 3(a) shows an exponential decay at low density, $\rho \leq 0.2$, with the decay time increasing with the concentration. At the higher densities, the decay changes dramatically, so the particles' direction of motion is getting much more stable in time. The spatial velocity correlation function, $C_{\parallel}(r)$, in Fig. 3(b) shows two distinct types of behavior: the decay is exponential at the two lowest concentrations, $\rho = 0.1$ and 0.2 , while it becomes algebraic at $\rho > 0.2$. We previously observed the transition to the power law form for two-point velocity correlations for the Vicsek-type model [39].

The cluster statistics for the density series is shown in Fig. 4. The plot in the inset confirms our observation that the cluster size is growing fast with the concentration. At $\rho = 0.05$ and 0.1 the cluster size distribution, as shown in the main plot, decays exponentially while at larger density ρ it changes into a power law, which has been observed previously and is characteristic for the ordered phase [39–41].

Now, we will look at the behavior of the system at constant density $\rho = 0.2$ while varying the energy influx rate q . As the ABP system changes the behavior from dissipative to driven upon increase of the energy influx rate, we expect the disordered motion at low q and onset of ordered behavior at high q levels. Fig. 5 shows the alignment of particles velocities at different input powers of the motor. At low q ($q = 0.2$, Fig. 5(a)) we see a practically homogeneous disordered system. Then, at $q = 0.5$ (Fig. 5(b)) well-shaped clusters are formed, within which the particles move in nearly the same direction. At high q , Fig. 5(c), the velocities are high, the clusters are compact, and we observe a significant degree of alignment. Note also that in the system with $q = 10$ there are very few single particles and most particles belong to the swarm. Note also the shape of the swarm: the ABP with dissipative interactions tend to form bands, which are perpendicular to their velocity.

Analysis of the velocity autocorrelation function in our model at fixed concentration $\rho = 0.2$ (Fig. 6(a)) shows that at $q > 0.3$ the motion of particles is very persistent. The change of the direction of motion is realised only through collisions between different clusters. However, we see a fast decorrelation of the velocity at low q , $q = 0.1$ and 0.3 , due to the thermal noise. In the spatial velocity correlations (Fig. 6(b)) we see a sharp transition from the exponential decay at $q = 0.1$ and 0.3 to a power law decay for larger q . All the curves showing the power law decay are practically identical.

Figure 7 presents the cluster statistics for the same density $\rho = 0.2$. The main plot shows the cluster size distribution on a log-log scale. We observe two qualitatively different distributions: at low energy influx rates,

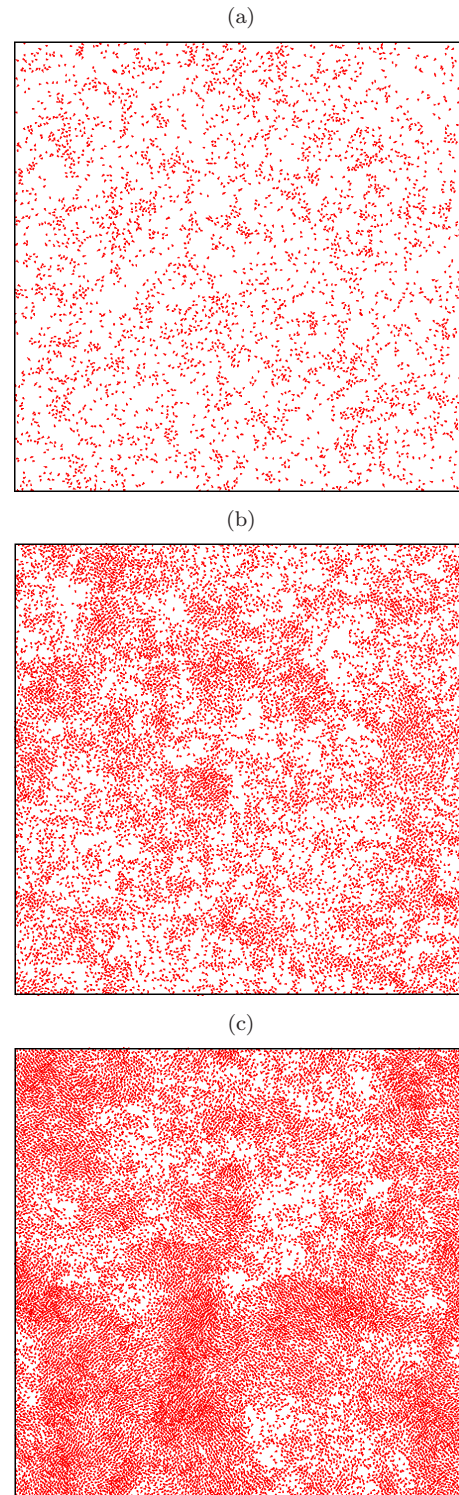


FIG. 2: Typical distribution of particles inside a simulation box at constant propulsion power $q = 0.3$ and different particle number densities: (a) $\rho = 0.1$, (b) $\rho = 0.3$, (c) $\rho = 0.5$.

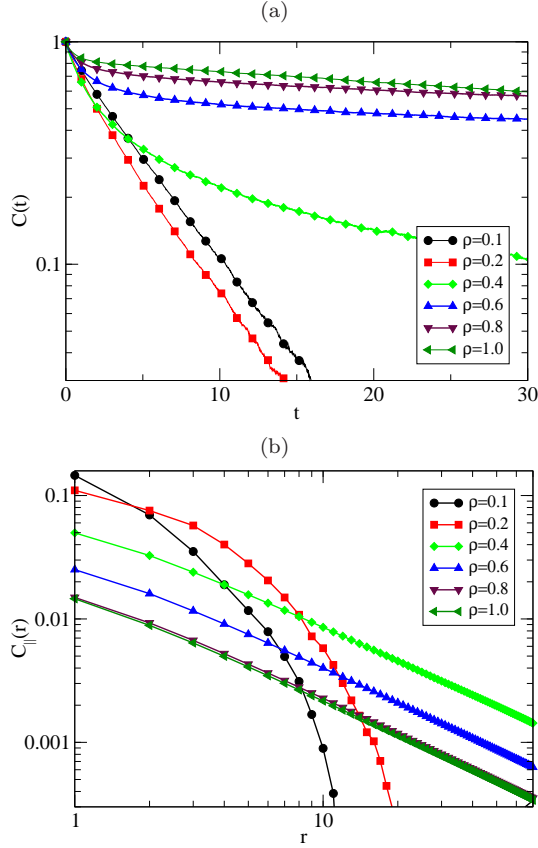


FIG. 3: Statistical properties of the ABP-DPD model at a constant energy influx rate $q = 0.3$. (a) Semi-log plot of velocity autocorrelation function $C(t)$ over time t . (b) Spatial velocity correlation function $C_{||}(r)$.

$q = 0.1$ to 0.3 , the curves show an exponential decay. At the higher q all of them are practically identical and have a straight segment at large numbers, which indicates the power law decay of the distribution. The transition can be located on the inset, where the evolution of the mean cluster size is shown. We see a kink on the curve at $q \approx 0.4$.

We measured the decay exponent for the spatial velocity correlation function, $C_{||}(r)$, in the whole range of studied parameters ρ and q and found that the exponent assumes universal values that depend only on the density but not on q , friction coefficients or temperatures. The values of the exponent are plotted in Fig. 8.

B. Orientational ordering

Behaviour of the order parameter at various densities is plotted on Fig. 9a. At low densities the order parameter values are close to zero during the whole simulation time which means that particle velocities are globally disaligned. At small propulsive power, $q = 0.3$, the ordering sets in slowly and reaches high values of about $\varphi \approx 0.8$ only at overlap densities of $\rho \approx 1$. At very high driving

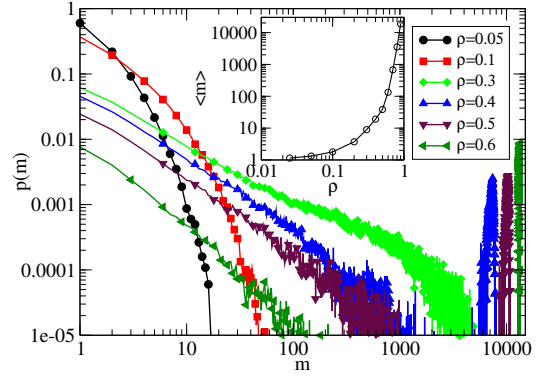


FIG. 4: Cluster statistics for the ABP-DPD model at a constant energy influx rate $q = 0.3$. *Inset:* The average cluster size. The exponent $p(m) \propto m^{-\zeta}$ for the straight segment: $\rho = 0.3$ $\zeta \approx 0.94$, $\rho = 0.4$ $\zeta \approx 1$, $\rho = 0.5$ $\zeta \approx 1.1$, $\rho = 0.6$ $\zeta \approx 1.3$.

power, $q = 100$, the order parameter reaches unity at densities of about $\rho = 0.04$ that corresponds to the mean distance between the particles is $r = \rho^{-1/2} = 5$, which is greater than the radius of interaction $r_c = 3$. Obviously, the cohesive effect of the collisions keeps particles together, as can be seen already from the snapshots in Fig. 5.

The phase diagram for our system is shown on Fig. 10. The location of the transition points for each set of parameters was determined using the standard Binder cumulant analysis from the intersection of the cumulant curves G_L calculated for three different system sizes. The behavior of the cumulant indicates the continuous character of the transition. It is clearly seen that the ordered behavior, at fixed environmental noise, is possible at certain minimum energy influx rate q , which, in its turn, determines the average propulsion speed. The critical energy influx rate changes with concentration according to the power law $q_c \propto \rho^{-\kappa}$, where κ is 0.46 ± 0.02 . We show the transition lines for two ambient temperatures, $T^E = 0.3$ and 0.6 . The twice as higher temperature of the environment at fixed friction γ^E means that the passive fluctuations (D^E) are twice as more intense and a higher energy influx is required for the ABP to be able to align. The q_c values required for the transition at $T^E = 0.6$ and $T^S = 0$ are roughly 1.4 times higher than those found at $T^E = 0.3$ and $T^S = 0$. The q_c values observed $T^E = T^S = 0.3$ are very close to those obtained at $T^E = 0.6$ and $T^S = 0$.

Here, we would also like to demonstrate how the particle individual and collective dynamics depends on the key parameters of the interaction. As mentioned above, the velocity correlations in our system decay exponentially in time in both phases according to $C(t) \propto e^{-t/\tau^S}$ (see Fig. 11). We have measured the correlation time τ^S to demonstrate the role of the intraswarm dissipation, which is controlled by γ^S . In a system without interactions, the relaxation time would be completely deter-

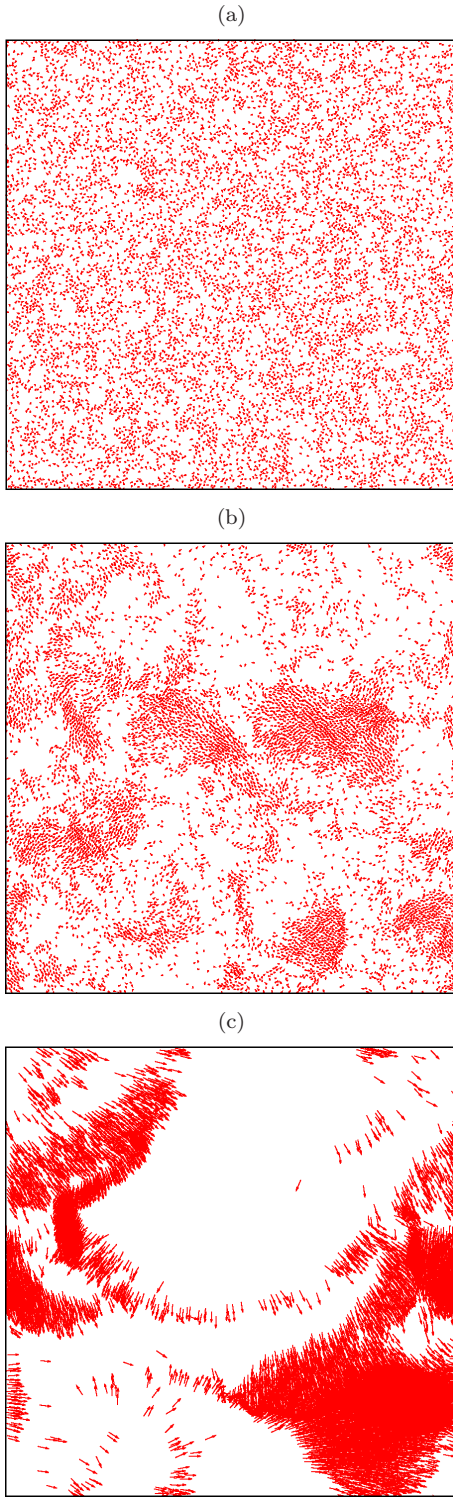


FIG. 5: Typical distribution of particles inside simulation box at constant density $\rho = 0.2$. (a) $q = 0.2$, (b) $q = 0.5$, (c) $q = 10$.

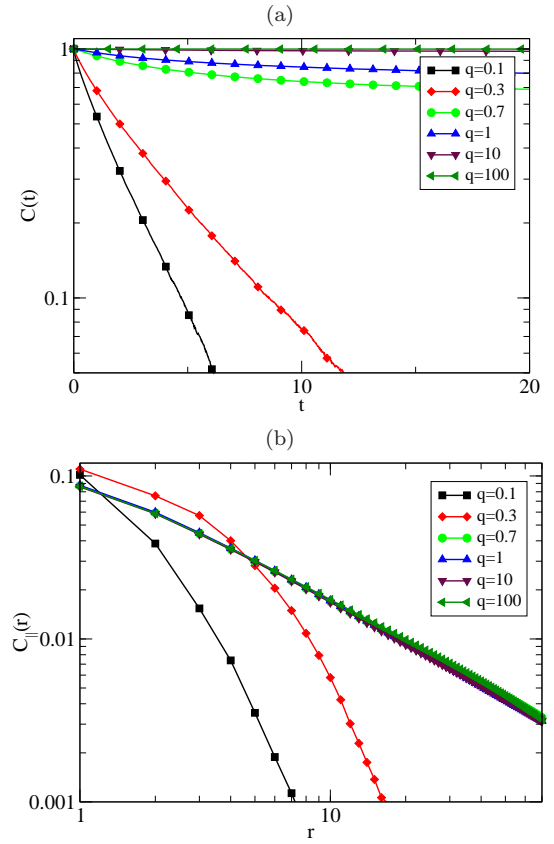


FIG. 6: Statistical properties of the ABP-DPD model at constant density $\rho = 0.2$. (a) Semi-log plot of velocity auto-correlation function $C(t)$ over time t . (b) Spatial velocity correlation function $C_{||}(r)$.

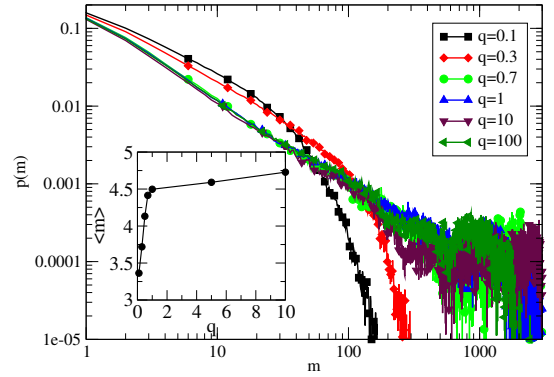


FIG. 7: Cluster statistics for ABP-DPD model at constant density ($\rho = 0.2$). *Inset*: Average cluster size. The exponent $p(m) \propto m^{-\zeta}$ for the straight segment: $q = 0.7$ $\zeta \approx 1.16$, $q = 1.0$ $\zeta \approx 1.1$, $q = 5.0$ $\zeta \approx 1.1$, $q = 10.0$ $\zeta \approx 1.11$.

mined by the dissipative and driving mechanisms of the Langevin equation and would normally decrease with increasing the friction, $\tau^E = m/\gamma^E$. In contrast, as can be seen in the plot, the correlation time in the swarm, τ^S , is growing proportionally to γ^S . The friction coefficient γ^S scales the dissipative power of the pairwise collisions and

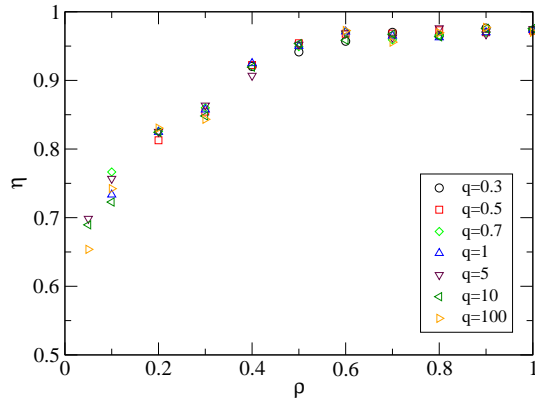


FIG. 8: Behavior of the exponent for the velocity correlation function, $C_{\parallel}(r) \propto r^{-d+2-\eta}$

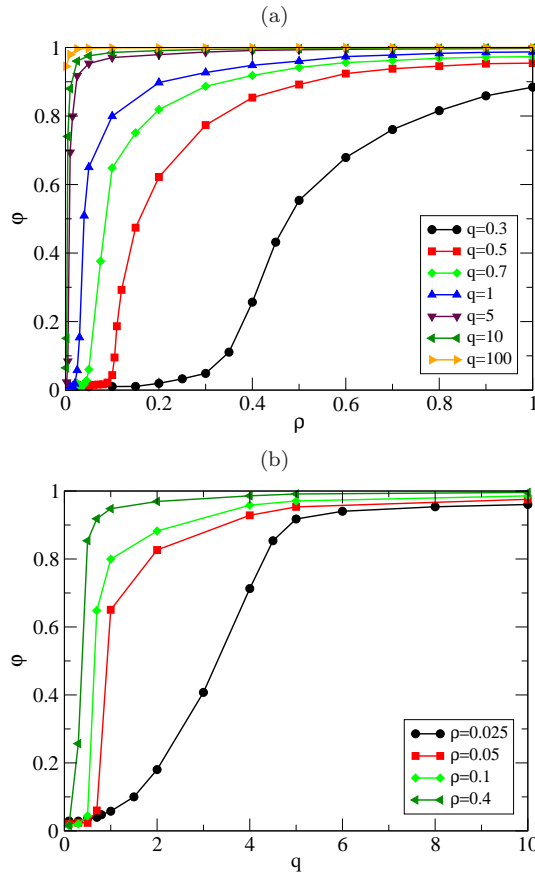


FIG. 9: Orientational order parameter for our model ($\varphi = 1$ corresponds to completely ordered system, $\varphi = 0$ - to completely disordered system).

therefore is the key parameter controlling the alignment. Moreover, in the main plot we see that the mean order parameter is also growing larger with γ^S .

Finally, Fig. 12 illustrates the role of the swarm temperature T^S . The swarm temperature in our model can be defined via a fluctuation-dissipation relation for the parameters of the pairwise interaction, noise and the fric-

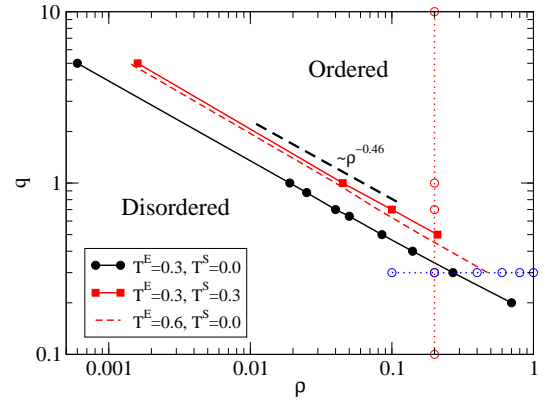


FIG. 10: Phase diagram for the ABP-DPD model at $\gamma^E = 0.3$, $\gamma^S = 1$, $T^S = 0$.

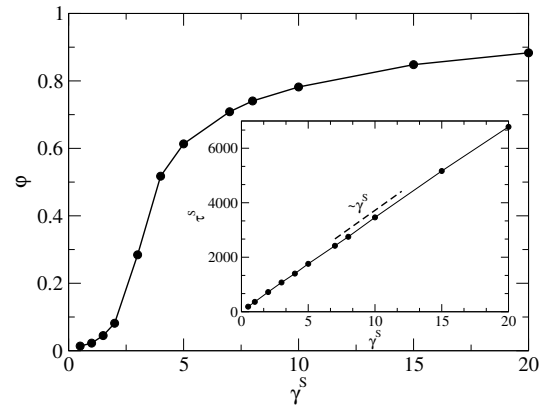


FIG. 11: Orientational order parameter in the ABP-DPD model as a function of intraswarm friction γ^S . *Inset:* Velocity correlation time vs friction γ^S ($T^S = 0$, $\rho = 0.2$, $q = 0.3$).

tion coefficient, as given by the Eq. (11). In terms of the temperature, the transition looks completely analogous to what is seen in the magnetic systems. At zero temperature, the ordering is maximal, while it is suppressed by the fluctuations and vanishes at certain maximal temperature T_c^S , which is increasing with the input power q . The order parameter approaches zero according to a power law $\phi \propto |T_c^S - T^S|^\beta$ with $\beta=0.52$ for $q = 0.5$, $\beta = 0.41$ for $q = 0.7$, and $\beta = 0.37$ for $q = 1$, which is in agreement with the critical exponent β reported earlier for the Vicsek model and other models with aligning interactions [41, 42].

IV. DISCUSSION

As we can see from the numerical data, a system of ABP with dissipative interactions indeed demonstrates the same qualitative properties as the well studied Vicsek model. In contrast to most previous approaches, here we have separated two influences on the particle motion: the effect of the environment, which is intro-

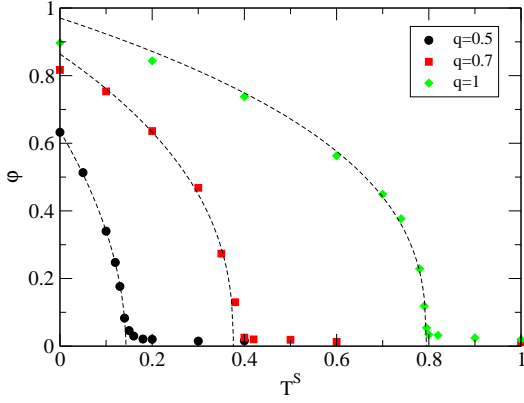


FIG. 12: Behaviour of the order parameter as a function of the swarm temperature T^S ($\gamma^S = 1$, $\rho = 0.2$, $T^E = 0.3$). The dashed lines show the power law fit to the points left of the transition temperatures.

duced through the Langevin equation (1), and the effect of the other active agents, where the interactions are set by its own pairwise dissipative parameter. Both effects can be associated with a temperature, a corresponding noise and a friction parameter that controls the rate of dissipation. Note that these two types of noise and dissipation have different influence on the system. The former one is acting even on single particles, while the latter applies only to the groups and automatically vanishes for single agents. At zero temperature of the environment, $T^E = 0$, the model reduces to the motion with a constant speed, usually referred to as self-propelled particles, as for instance in the Vicsek model. At $T^S = 0$, we have a system with aligning inelastic collisions but without the corresponding active noise. As we see from Fig. 11, the pairwise friction that scales the dissipation power in the collisions can be used to regulate the amount of alignment in the system, and, hence, the mean order parameter. Thus, our model allows one to mix these contributions in different proportions and model different swarming scenarios.

Now, we would like to discuss the extent of the differences and similarities of the swarming behavior in our model to observations from the Vicsek model in more detail. The main difference of the present analysis from the previous studies is that in most simulations we assumed a constant noise, as it is associated with the action of the environment, and varied the propulsive power of the particles. This path, however, can be conveniently mapped onto a situation with a fixed particle speed and a variable noise. In static isolated systems, the ratio of the characteristic interaction energy to the thermal energy completely determines the equilibrium state. In the swarm of active particles, the crucial number is the ratio of the stationary velocity due to propulsion to the characteristic velocity due to thermal fluctuations. This ratio can

be written as

$$\frac{V_0^2}{\langle V_{eq}^2 \rangle} \approx \frac{q}{\gamma^E} \frac{m}{k_B T^E} = \frac{q \tau^E}{k_B T^E} \quad (20)$$

where we used the relation between the friction and the relaxation time in the Langevin equation, $\tau^E = m/\gamma^E$, and the equipartition relation, $m \langle V_{eq}^2 \rangle = k_B T^E$. Thus, the ratio in question is equivalent to the incoming energy within the characteristic relaxation time, $q \tau^E$, to the thermal energy. In case the noise level is fixed by T^E , is it the stationary particle speed that matters. The mean speed in the ABP model at large q is given by $V_0^2 = q/\gamma^E$. At higher temperatures of the environment, one needs to pump in more energy to produce the same ratio of the characteristic speeds. This point is confirmed by the data presented in Fig. 10. In case the stationary speed is fixed, one needs to reduce the temperature, which is, in the Langevin or DPD models, proportional to the fluctuation amplitude D . Therefore, the phase diagram in terms of noise amplitude D vs density ρ or T^E vs ρ will be inverse of our diagram shown in Fig. 10. It is interesting to note that the sum of the critical swarm temperature, as shown in Fig. 12, and the ambient temperature T^E is roughly proportional to the energy influx rate. Here, we have $T_c^S + T^E \approx 0.2 + 0.3 = 0.5$ for $q = 0.5$, $T_c^S + T^E \approx 0.7$ for $q = 0.7$, and $T_c^S + T^E \approx 0.8 + 0.3 = 1.1$ for $q = 1$. So, the fluctuations of different nature simply add up to increase the effective swarm's temperature, which can be defined as $T = T^E + T^S$. This idea is supported also by the phase diagrams shown in Fig. 10, where two systems with equal values of $T^E + T^S = 0.6$ demonstrate a transition at nearly the same q and ρ .

In simulations, we observe aggregation and orientational ordering of ABP at sufficiently high densities in presence of sufficiently high propulsive power. At the fixed level of noise and propulsive power, the cluster size grows with the particle concentration in the same way as we observed previously for the Vicsek model [39]. Thus, the dissipative interactions as well lead to cohesion of active particles. Secondly, they lead to particle alignment as can be seen from the growth of the order parameter with concentration, again, similar to the dependence seen in the Vicsek model. The transition into the orientationally ordered phase happens along the line $q_c \propto \rho^{-0.46}$, which is an inverse of the transition line for the Vicsek model, where it happens at $\xi \propto \rho^{0.45}$, where ξ is the noise amplitude [39, 43]. This power law behavior seems to be not unique to the Vicsek model, but a universal property of systems with global alignment and has been reported also for systems with pairwise aligning interactions (with an exponent $\kappa = 0.46 \pm 0.04$) [26].

In what regards other properties, we should mention the behavior of the correlations functions $C(t)$ and $C_{||}(r)$ (Figs. 6 and 3), which demonstrate the same qualitative features as the Vicsek model we studied previously [39]. The two-point velocity correlation function changes the shape from exponential to a power law at the critical point and inside the whole region of the ordered behavior.

The exponent η , which describes the decay of $C_{\parallel}(r) \propto r^{-d+2-\eta}$ in the ordered phase, takes the same values from 0.5 to 0.97 on increasing density and shows the same density dependence as we previously saw in the Vicsek-type model. It seems to be insensitive to other details of the system and reflects just the nature of the ordering. At the transition point, the exponent is expected to satisfy the Fisher's scaling law: $\gamma/\nu = 2 - \eta$ [44], where γ and ν are the critical exponents for isothermal susceptibility and the fluctuation correlation radius. In the limit of low concentrations, where the repulsions are not important, we have $\eta = 0.5$, thus $2 - \eta = 1.5$, which is in agreement with the result for $\gamma/\nu = 1.47$ obtained previously for the standard 2D Vicsek model [42]. Moreover, the shape of the cluster size distributions as shown in Figs. 7 and 4 in our model is also identical to that for the Vicsek model ranging from $\zeta = 0.5$ to $\zeta = 1.5$ depending on the level of noise and the density [39–41]. Although the type of active particle, the interactions and the type of noise differ from those in the Vicsek model, the identical values of the exponents suggest that our model belongs to the same universality class [41].

V. CONCLUSIONS

We have studied dynamic self-organization in a model combining the active Brownian particles with dissipative

particle interactions, which are introduced via inelastic collisions. We find that the ABP-DPD model exhibits an orientational order-disorder transition on increasing energy influx rate or particle number density, which is completely analogous to that in the Vicsek model. We have shown that the parameter space of such an active system can be reformulated in terms of effective temperatures of the environment and of the swarm and the ratio of the characteristic thermal energy to the energy influx per particle.

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